

# Issues with vacuum energy as the origin of dark energy

*Houri Ziaepour*

Max Planck Institut für Extraterrestrische Physik (MPE),  
Giessenbachstraße, Postfach 1312, 85741 Garching, Germany.  
houriziaepour@gmail.com

March 4, 2013

## Abstract

In this letter we address some of the issues raised in the literature about the conflict between a large vacuum energy density, apriori predicted by quantum field theory, and the observed dark energy which must be the energy of vacuum or include it. We present a number of arguments against this claim and in favour of a null vacuum energy. They are based on the following arguments: A new definition for the vacuum in quantum field theory as a frame-independent coherent state; Results from a detailed study of condensation of scalar fields in FLRW background performed in a previous work; And our present knowledge about the Standard Model of particle physics. One of the predictions of these arguments is the confinement of nonzero expectation value of Higgs field to scales roughly comparable with the width of electroweak gauge bosons or shorter. If the observation of Higgs by the LHC is confirmed, accumulation of relevant events and their energy dependence in near future should allow to measure the spatial extend of the Higgs condensate.

## 1 Introduction

This work is an attempt to find an answer for the following question:

*Can the observed dark energy be due to a small nonzero vacuum energy ?*

This question is not simple to answer. We need to have a precise definition of what we call *vacuum*. Remind that in quantum field theory both an empty state i.e. a state with no particle and the minimum of the potential are called *vacuum*. Because energy is the source for gravity, a detailed examination of the nature of vacuum and its energy density apriori needs a quantum gravity model and a knowledge of the quantum state of the Universe, at least from the earliest times relevant for present observations, presumably since inflation epoch. String theory, which for the time being is the most popular quantum gravity candidate, allows some  $\sim 10^{500}$  *vacuum* states, generated by the compactification of internal dimensions. In effective field theory models they correspond to the minima of moduli potential, thus they are not *empty states*. For the time being there is no generally accepted selection rule or probability distribution for this landscape. Considering a more phenomenological setup, for instance an empty Minkowski spacetime at infinite past, quantum fluctuations can create pair of particles. This process is irreversible because the presence of particles modifies the geometry of the spacetime and the probability of self-annihilation, and leads to a non-empty state. Therefore, it seems that when gravitational effects are considered, a true *vacuum* - empty space - cannot exist and is only an abstract concept.

In a few recent reviews of dark energy models [1, 2, 3] authors have argued in favour or against an small but nonzero value of vacuum energy density. The present letter can be classified in the latter category i.e. here

we want to argue that based on fundamental principles of quantum mechanics, vacuum energy must be zero. Notably, we show that the Casimir effect which is usually used as an evidence for gravitational interaction of vacuum [9] is related to close relation between quantum mechanics, symmetries, and observables. After renormalization these effects are included in the effective mass and couplings of particles and the vacuum itself can be considered as being an empty space without any energy.

Another issue that is claimed to be problematic for quantum field theory is the large vacuum expectation value of known or expected condensates in the framework of Standard Model. In particular, quark-antiquark condensate  $\langle \bar{q}q \rangle \neq 0$  responsible for the breaking of chiral symmetry in QCD [4, 5, 6] and Higgs condensate which is necessary for generation of the mass of electroweak gauge bosons [9, 2, 3] have been the focus of a large number of studies. Apriori one of the conditions for the formation of condensates is the extension of condensate field to an infinite volume with roughly the same amplitude. Apparently, the application of this rule would mean that the Standard Model condensates should contribute to the cosmological vacuum energy term in the Einstein equation and should make it some  $10^{40}$  or more folding larger than the observed dark energy density. Therefore, the small observed value for dark energy is in strong contradiction with this expectation, and apriori one could consider this as the signature of the failure of quantum field theory. However, recently various arguments have been suggested to prove that in the case of quark-antiquark condensate, the nonzero expectation value  $\langle \bar{q}q \rangle \neq 0$  is confined into hadrons volume, and therefore its effect is included in their mass and does not contribute to dark energy [7]. Here we extend these arguments to Higgs and other scalar fields expected in the extensions of the Standard Model such as SUSY. We show that they cannot have an effect on the vacuum energy of the Universe at large scales. Thus, the origin of dark energy must be searched elsewhere, notably in the condensation of one or multiple light or preferentially massless quantum fields - to prevent the need for fine-tuning.

## 2 Vacuum energy in quantum field theories

We begin with a review of the definition of what is called a *vacuum*. As everybody knows, the dictionary definition of this word is *emptiness*, the absence of any material. This is exactly the way it is used in the context of classical physics. However, in quantum field theory its meaning is more subtle, and for this reason, in the literature its exact definition is somehow context dependent. For instance, consider the case of bosonic fields. The Fock space is generated by creation and annihilation operators defined according to the following decomposition:<sup>1</sup>

$$\phi(\vec{x}, t) = \sum_{\{\alpha\}} u_{\{\alpha\}}(\vec{x}, t) a_{\{\alpha\}} + u_{\{\alpha\}}^*(\vec{x}, t) a_{\{\alpha\}}^\dagger \quad (1)$$

$$[a_{\{\alpha\}}, a_{\{\alpha'\}}] = 0, \quad [a_{\{\alpha\}}^\dagger, a_{\{\alpha'\}}^\dagger] = 0, \quad [a_{\{\alpha\}}, a_{\{\alpha'\}}^\dagger] = \delta_{\{\alpha\}\{\alpha'\}} \quad (2)$$

where  $u_{\{\alpha\}}$  and  $u_{\{\alpha\}}^*$  are solutions of dynamic equation for the set of parameters  $\{\alpha\}$  in an arbitrary spacetime. The vacuum state is defined as:

$$a_{\alpha} |0\rangle = 0, \quad \forall \alpha \quad (3)$$

Through this work we use this definition for the vacuum. The decomposition (1) and definition (3) are usually performed for free fields, thus in presence of interactions this decomposition is valid only if the model is perturbative. For instance, in flat Minkowski spacetime  $\{\alpha\} = \{k^0, \vec{k}\}$ ,  $u_k = e^{-ikx}$  and  $k^{02} = \vec{k}^2 + m^2$  for on-shell modes. This definition uniquely determines the vacuum, up to an arbitrary Lorentz transformation of the reference frame.

---

<sup>1</sup>Through out this work we assume  $\hbar = c = 1$ .

For a free classical scalar field the energy density - the 00 component of energy-momentum tensor - is  $T^{00} = 1/2\partial^0\phi\partial^0\phi + 1/2m^2\phi^2$ . For a quantum scalar field the same expression is usually used but the classical field is replaced by the quantum field operator. Then, the decomposition (1) is applied to determine the vacuum expectation value of energy density  $\langle 0|\hat{T}^{00}|0\rangle$ . Restricting ourselves to on-shell modes we find:

$$\langle 0|\hat{T}^{00}|0\rangle = \langle 0|\frac{1}{(2\pi)^3} \int d^3k u_k u_k^* \omega_k (a_k a_k^\dagger + a_k^\dagger a_k)|0\rangle = \frac{1}{(2\pi)^3} \int d^3k \omega_k \rightarrow \infty \quad \omega_k = \sqrt{\vec{k}^2 + m^2} \quad (4)$$

The infinite or very large value of  $\langle 0|\hat{T}^{00}|0\rangle$  - when a cutoff to UV limit of integral in (4) is introduced - is the famous problem of vacuum energy. In curved spacetimes expressions for  $u_{\{\alpha\}}$  and  $u_{\{\alpha\}}^*$  are usually more complicated and in fact exceptionally analytic expression for them exist. The energy  $\omega_\alpha$  is also in general more complicated and depends on time. In some cases such as Anti-de Sitter spacetime, after renormalization a finite vacuum expectation value is obtained [8] which is inversely proportional to the volume of the spacetime.

The origin of the singularity in (4) is well understood [10]. It is due to the fact that in quantum field theory, operators  $\phi^2$  and  $(\partial^0\phi)^2 \equiv \Pi^2$  in  $\hat{T}^{00}$  are not well defined and must be replaced by ordered operators:

$$\phi^2(x) \rightarrow :\phi^2(x): \equiv \lim_{y \rightarrow x} \left\{ \phi(x)\phi(y) - \langle 0|\phi(x)\phi(y)|0\rangle \right\} \quad (5)$$

$$\Pi^2(x) \rightarrow :\Pi^2(x): \equiv \lim_{y \rightarrow x} \left\{ \Pi(x)\Pi(y) - \langle 0|\Pi(x)\Pi(y)|0\rangle \right\} \quad (6)$$

This operation brings the vacuum energy to zero, as expected for an empty space, and is one of the regularization methods used in quantum field theories. There are other regularization and renormalization methods too, some of them more suitable for applications in curved spacetimes, see e.g. [10] for a review. Nonetheless, generally it is considered that the application of regularization and renormalization techniques to energy-momentum tensor is ad hoc [1]. Therefore in a mysterious way, vacuum energy of various components must cancel each others to make the energy density of the vacuum consistent with observations, see [9] for a review. Another solution to this problem associates the small observed value of dark energy to one of many possibilities offered by the very large landscape of string theories [11, 12]. But, giving the fact that no generally accepted probability distribution on the string landscape exists, the value of vacuum energy density must have an anthropological bias [13, 14], i.e. universes with large vacuum density cannot develop galaxies, stars, and life.

## 2.1 New interpretations

Now let's go back and look through the problem from another angle. Mode functions  $u_k$  and  $u_k^*$  are solutions of the field equation which is a differential equation. They include arbitrary integration constants that are determined from initial or boundary conditions. In Minkowski spacetime conjugate functions  $u_k$  and  $u_k^*$  are independent solutions of the field equation, thus there is no ambiguity in the definition of particles and anti-particles. This is due to the fact that in Minkowski space there is a Killing vector for the whole space, and thereby there is a *natural* vacuum definition. However, in expanding spaces such as FLRW and de Sitter there is not a unique *natural* vacuum and  $u_k$  and  $u_k^*$  define vacua which are only approximately similar to Minkowski - adiabatic - vacuum. Various vacua are related to each other by a Bogolubov transformation, and correspond to adiabatic vacua in frames moving with respect to each other, not necessarily with constant velocity. The vacuum state in one frame is a non-vacuum state with infinite number of particles in another frame. According to general relativity there is no preferred frame, thus there is no preferred vacuum either, see e.g. [10] and references therein for more details. In the next section we propose another definition for vacuum in the context of quantum field theories. We believe it presents better the properties of what is

called *vacuum* in quantum field theories in which vacuum is very far from being an static empty space. Before presenting the new definition, we first discuss the physical interpretation of singularity in equation (4).

Consider the operator  $a_k a_k^\dagger + a_k^\dagger a_k$  in (4). The second term is the number operator  $\hat{N}_k$  and its eigen value is the number of particles in mode  $k$ . By definition the vacuum state  $|0\rangle$  is an eigen state of  $\hat{N}_k \forall k$  with eigen value zero. Thus, its application does not modify the vacuum. Assuming that an unambiguous notation of particle and anti-particle exists or defined by convention with respect to a given frame, the second term in the expression above can be decomposed to creation of one particle in mode  $k$  and then its annihilation. In a classical view these two operations are opposite to each other and should leave the space unchanged - specially when gravity and change of geometry due to particle creation discussed in the Introduction is not considered. Therefore, the constant term due to noncommutation of  $a_k$  and  $a_k^\dagger$  can be considered as a *remnant* energy. One interpretation is that this remnant is due to an error aroused from using the classical expression for  $T^{\mu\nu}$  which includes ambiguous operators. In this case the operator ordering or other regularization schemes are legitimate, and produce the correct expression for  $T^{\mu\nu}$  in the context of quantum field theories.

We can also interpret the remnant from a purely quantum mechanical point of view. The expression (1) can be decomposed in real space to creation and annihilation operators at the point  $x$  rather than in Fourier space. This simply corresponds to separately sum over all creation and annihilation terms. Then, the decomposition can be interpreted as the following: Because we know exactly the position of the created (annihilated) particle, we can have no information about its momentum which can be any value including infinity, and thereby the singularity of (4). In addition, we note that creation of a particle at a specified point breaks the translation symmetry - Poincaré group - of the spacetime, specially if we consider its gravitational effect on the spacetime. In fact even without creation of any particle the symmetry of the spacetime breaks by just associating an origin to it. The reason is again the Heisenberg uncertainty rules. Singling out one point of the spacetime induces infinite uncertainty in energy and momentum measurements, thus singularity of (4)<sup>2</sup>. In this interpretation regularization of the integral in (4) by imposing an energy cutoff is equivalent to introducing an uncertainty on the position measurement. In the literature usually the high energy cutoff is associated to the high energy physics. Here we see that according to this interpretation it presents the highest energy scale or equivalently smallest distances in which the observer can verify the presence of a vacuum. This minimum distance depends on the strength of  $\phi$  field couplings, stronger the coupling longer the distance. This brings us to the issue of the definition of vacuum and how its presence can be tested. We discuss this in the next section. Meanwhile, we should emphasis that the regularizing cutoff only provides an upper limit on the vacuum energy density and does not mean that the latter is nonzero. In this interpretation the effect of high energy modes are already considered in the effective mass of the particle and should not be considered. Note also that these interpretations do not take into account the arbitrary integration coefficients which are implicitly included in  $u_k$  and  $u_k^*$ . Neither we used explicit expressions for these functions which are expected to depend on the geometry of spacetime. Therefore, these arguments are valid for any vacuum and any spacetime.

## 2.2 Vacuum as a coherent state

According to the definition of vacuum in equation (3), there is no particle in a vacuum state in the frame for which it has been defined. Therefore, we expect no effect on a particle that passes through it. This means that such an experience can be used to test the presence of a vacuum. However, we know that quantum fluctuations affect a particle and after renormalization its effective mass for an observer at rest in

---

<sup>2</sup>These arguments are in spirit the same as the arguments given by Eppley & Hannah [15] and in [16] to prove the inconsistency of a classical gravity and a quantic matter.

the vacuum frame depends on its energy. Therefore, it is the sensitivity of an observer to energy variation that determines how well the vacuum can be detected. This interpretation is consistent with our claim in the Introduction that vacuum is an abstract concept and in the context of quantum field theory its presence is always an approximation. Considering the complexity of the vacuum and the fact that it is not really the no-particle state that its definition pretend, here we suggest a new definition for it.

In [18] we described a generalized coherent state for a scalar field based on an original suggestion by [17] as:

$$|\Psi_{GC}\rangle \equiv \sum_k A_k e^{C_k a_k^\dagger} |0\rangle = \sum_k A_k \sum_{i=0}^{N \rightarrow \infty} \frac{C_k^i}{i!} (a_k^\dagger)^i |0\rangle \quad (7)$$

$$a_k |\Psi_{GC}\rangle = C_k |\Psi_{GC}\rangle \quad \langle \Psi_{GC} | N_k | \Psi_{GC} \rangle = |A_k C_k|^2 \quad (8)$$

For  $\{C_k \rightarrow 0 \forall k\}$  this state is neutralized by all annihilation operators and the expectation value of particle number approaches zero for all modes. Therefore, this state satisfies the condition (3) for a vacuum state. Coefficients  $A_k$  are relative amplitude of modes  $k$  and can be nonzero even for vacuum state. In addition, one can extend this definition by considering products of  $|\Psi_{GC}\rangle$  states. Such a state includes products of states in which particles do not have the same momentum. Thus, it consists of all combinations of states with any number of particles and momenta.

$$|\Psi_G\rangle \equiv \sum_{k_1, k_2, \dots} \left( \prod_{k_i} A_{k_i} \right) e^{\sum_i C_{k_i} a_{k_i}^\dagger} |0\rangle \quad (9)$$

In this case all  $C_{k_i} \rightarrow 0 \forall k_i$  in a vacuum state. Under a Bogolubov transformation this state is projected to itself:

$$a_{k_i} = \sum_j \sum_{k_j} \mathcal{A}_{k_j k_i} a'_{k_j} + \sum_j \sum_{k_j} \mathcal{B}_{k_j k_i} a'^{\dagger}_{k_j} \quad a'^{\dagger}_{k_i} = \sum_j \sum_{k_j} \mathcal{A}^*_{k_j k_i} a'^{\dagger}_{k_j} + \sum_j \sum_{k_j} \mathcal{B}^*_{k_j k_i} a'_{k_j} \quad (10)$$

Replacing  $a_{k_i}^\dagger$  in (9) with the corresponding expression in (10) leads to an expression for  $|\Psi_G\rangle$  similar to (9) but with respect to the new operator  $a'^{\dagger}_k$  and  $C'_{k_j} = \sum_i \sum_{k_i} \mathcal{A}^*_{k_j k_i} C_{k_i}$ . For  $C_{k_i} \rightarrow 0 \forall k_i$  and finite  $\mathcal{A}^*_{k_j k_i}$ ,  $C'_{k_i} \rightarrow 0 \forall k_i$ . Note that here we assume that the Bogolubov transformation changes  $|0\rangle$  to a similar state which is neutralized by  $a'_k \forall k$ . Therefore, in contrast to the null state of the Fock space,  $|\Psi_G\rangle$  is frame-independent. However, it is easy to verify that this new definition of vacuum does not solve the problem of singular integral when one tries to determine the expectation value of  $\hat{T}^{00}$  without operator ordering, because as explained above this issue is related to the definition of  $\hat{T}^{00}$ . Nonetheless, it gives a better insight into the nature of the problem. Notably, one can use the number operator  $\sum_k \hat{N}_k$  to determine the energy density of vacuum because in contrast to  $|0\rangle$ , the new vacuum  $|\Psi_G\rangle$  is frame-independent and is neutralized by the number operator  $\hat{N}_k |\Psi_G\rangle = 0 \forall k$ . This alternative to  $\hat{T}^{00}$  for measuring the vacuum energy density has been discussed in [10], but has been considered to be a poor replacement because vacuum state in one frame can be a state with nonzero number of particles in another frame. Consideration of  $|\Psi_G\rangle$  as vacuum has the advantage that it is invariant under Bogolubov transformation. Thus, if  $\sum_k |A_k C_k|^2 \rightarrow 0$  in one frame it is null in all frames. If a state do not have any particle for all observers, its energy density must be zero. This frame-independent argument proves that the application of regularization when one uses  $\hat{T}^{00}$  for the same purpose is not an ad hoc operation.

Coefficients  $A_k$  should be calculated from the full Lagrangian of the model using usual quantum field theory techniques. Evidently, the full solutions of propagators which is necessary for determining  $A_k$ 's depend on the initial or boundary conditions. Nonetheless, as  $|\Psi_G\rangle$  contains all possible states, different initial conditions project it to itself. In this sense this state is unique. Moreover, as  $|\Psi_G\rangle$  is a maximally coherent state and its member states have vanishing amplitude, they are not directly observable. Similar to a usual

entangled state, one can say that vacuum *collapses* to one of its member states when it is observed i.e. in the process of interaction, for instance with an on-shell particle. But because any single state has a vanishing amplitude, one can always consider the vacuum unchanged even when one or any finite number of its members interact with an untangled state, and thereby is indirectly observed as virtual particles. Their effect manifests itself as scale dependence of mass and couplings of the field. Thus, like usual definition of vacuum, interactions modify properties of the external (untangled) particles at scales relevant for the interaction, but they don't change  $|\Psi_G\rangle$ <sup>3</sup>.

Note that  $|\Psi_G\rangle$  includes all states at any scale. However, in every experiment only a limited range of scales are available to observers. They are limited from IR side by the size of the apparatus or observational limits such as a horizon, and from UV side by the available energy to the observer. The presence of a particle at a given scale i.e. discrimination between vacuum and non-vacuum at that scale depends on the uncertainties of distance/energy measurements. At large distance scales the limited sensitivity of detectors cannot detect interaction of untangled particles with very low energy virtual particles and no violation of energy-momentum conservation occurs. This couldn't be true if the vacuum had a large energy-momentum density which could be exchanged with on-shell particles at any distant scale.

Does the coherent vacuum state  $|\Psi_G\rangle$  gravitate ? A detailed answer to this question needs a quantum description for gravity and what a vacuum means in this context. Nonetheless, even without knowing the details of the vacuum state  $|\Psi_G\rangle$  for quantum gravity, by definition states that make up this coherent state are not observable except when they are decohered/collapsed. And when this happens, they will not appear as vacuum. Consequently, they cannot influence observations in any way including gravitationally. In a semi-classical view, one expects that the expectation number of particles with a given energy and momentum determines the strength of gravitational force. Equation (8) shows that this number for any value of energy and momentum is null when  $\{C_k \rightarrow 0 \ \forall \ k\}$ . Thus this state does not feel the gravity. This shows the unphysical nature of energy-momentum tensor singularity when its classical definition is used in quantum field theory without regularization.

A good example of such vacuum is the ensemble of electrons in valance energy levels inside a solid. Although electrons are fermions and their number in each energy level is restricted to 2, this system presents very well the properties of the vacuum state discussed above. At zero temperature all the states under Fermi level are filled and pair of electrons make an entangled ensemble. Injected external - on shell - electrons or photons can decohere e.g. one electron and make a pair of electron-hole - an exciton [19]. But the vacuum stays (approximately) unchanged. If the injected energy does not significantly increase the temperature, the lifetime of the electron-hole pair (exciton) will be short, and they recombine quickly. This is similar to vacuum fluctuation of fundamental quantum fields<sup>4</sup>. The energy necessary for the creation of electron-hole pair is provided by the exciting agent, a photon or another electron, and the effect can be considered as a change in the kinetic energy of the colliding photons - usually from a laser source. Therefore, creation of virtual particles do not affect the interaction of the system with gravity (also see Sec. 3). This example shows that the state  $|\Psi_G\rangle$  unifies the definition and presentation of vacuum state in particle physics and in condense matter physics. Moreover, because it has the same form as a coherent condensate state, it also put the concept of vacuum as a state without particle and vacuum as the minimum of the potential of a condensate into the same formal formulation. This unified description is notably interesting in the context of symmetry breaking and phase transition. We leave a more extended study of  $|\Psi_G\rangle$  and its properties to a future work.

---

<sup>3</sup>In the interaction picture states are evolved according to free Hamiltonian. Therefore, interactions do not modify  $|\Psi_G\rangle$ .

<sup>4</sup>Note that similar to production of Cooper pairs of electrons in superconductors, the pairing of electron-hole and formation of the exciton pseudo-scalar field occurs only at low temperatures. At higher temperatures electrons and holes behave independently as fermionic gases.



## 2.3 Vacuum symmetry

The issue of the vacuum energy can be considered yet in another way. When operator  $a_k^\dagger$  is applied to the vacuum, one particle with momentum  $\vec{k}$  is produced. But, because we exactly know its momentum, there is no information about its position. Therefore, an observer who wants to apply the annihilation operator must first somehow localize the particle, otherwise the probability of annihilation becomes negligibly small. Such an operation would not be possible without breaking the translation symmetry of the spacetime, for instance by imposing boundaries at a distance  $L \sim 1/k$ . This induces a Casimir energy proportional to  $1/L \sim k$ , thus infinite energy for  $k \rightarrow \infty$  i.e. if we want to be sure that the particles is annihilated.<sup>5</sup> This description and the definition of vacuum as a coherent state given in the previous section are different view of the same reality, i.e. the close relation between quantum mechanics, its non-locality, and symmetries. The effect of particle creation given above explicitly shows the manifestation of this non-locality. Therefore, introduction of ordering operator or any other regularization-renormalization prescription can be considered as a shorthand for the operation described above i.e. creation of a particle with a given momentum in the Minkowski spacetime, limiting the space which needs/produces energy, annihilating the particle, and finally removing boundaries which produces/needs energy<sup>6</sup>.

Some authors [9] have considered the infinity or very large vacuum energy obtained in (4) as a physical reality, specially because of the observation of Casimir effect [21, 22]. Thus, they have tried to find a way to neutralize vacuum energy by considering supersymmetry or quantum cosmology (see [9] for review and references therein). Casimir effect is observable when the symmetry of an empty spacetime is explicitly broken, for instance by introducing a boundary which divides the space. As we have argued above, any symmetry breaking operation changes the quantum state of the spacetime, notably from a vacuum to a *non-vacuum*. One can define a vacuum as the state with smallest information - entropy. This is a direct conclusion of definition (7). Every symmetry breaking induces some information. In the example above the division of space in two parts creates information because modes/particles are assumed to belong to one or the other part not both. Moreover, the confinement of particles in space is not a trivial operation and a *wall* or potential - thus energy - is needed to prevent particles to penetrate to the other part(s). This close relation between the way an experiment is performed and its outcome is a well known concept in quantum mechanics. Furthermore, inconsistencies of a semi-classical treatment of gravity are well known [28, 16]. A very explicit conflict between two theories, which is also directly related to the subject of vacuum energy, is the fact that by defining an absolute energy reference we lose all information about the position of the reference object.<sup>7</sup>

Summarizing the conclusion of arguments given in this section, the regularization of energy-momentum tensor in quantum field theory is a necessary and legitimate technique for obtaining a physically meaningful value for this quantity. Accepting this point, it would be interesting to have a quantitative estimation of the energy density of vacuum after regularization for some physically important spacetimes. It has been shown [10] that renormalized energy momentum tensor  $\langle T^{\mu\nu} \rangle_{ren}$  in de Sitter space is  $\propto R_{dS}^{-4}$  where  $R_{dS}$  is the radius of the de Sitter space. Assuming an initial radius  $\sim H_{inf}^{-1}$  for an  $\alpha$ -vacuum,  $H_{inf} \lesssim 10^9$  GeV,

---

<sup>5</sup>This argument also shows that the Casimir effect [21, 22] produced by small anisotropies only add a finite contribution to the infinity obtained in (4). In fact, it adds a random component, because fluctuations are random. Therefore it cannot explain the presence of a uniform dark energy.

<sup>6</sup>For the sake of simplicity here we only discussed the case of a Minkowski spacetime. Our arguments applies equally to other spacetimes, notably FLRW and de Sitter which are important in the cosmological context. If an argument is specific to one of these spacetimes, it is explicitly reminded in the text.

<sup>7</sup>Note that this argument is not in conflict with  $E_{min} = 0$  in supersymmetric models. The statement in these models is that the potential energy of all particles and their supersymmetric partner is zero. Although one can conclude that in SUSY models it is not possible to have a negative energy, in what concerns measuring the potential the laws of quantum mechanics are applied, i.e. if we want to verify  $E_{min}$  theorem by measuring its value with very good precision, we lose information about the position of particles.

and  $\gtrsim e^{60}$  folds expansion, the vacuum energy at the end of inflation must be  $\lesssim 10^{-32} \text{ eV}^4$  which is much smaller than the observed energy density of dark energy  $\rho_{de} \sim 10^{-11} \text{ eV}^4$ .

In a similar way, one can formulate and apply regularization-renormalization to the expectation value of energy density in other - less symmetric - curved spacetimes such as FLRW. Dynamic equation in FLRW do not always have an analytical solution, but approximate solutions have been found in [18]. In particular, in flat-FLRW geometry equal-time space-like surfaces have the same geometry as their counterpart in Minkowski space. Thus, crossing singularity and its regularization has the same properties too. The dominance of dark energy and accelerating expansion of the Universe at present makes the spacetime to approach to a de Sitter geometry. Thus, using the same relation between  $\langle T^{\mu\nu} \rangle_{ren}$  and horizon size  $\sim H_0^{-1}$  should give an acceptable order of magnitude estimation for an accelerating FLRW geometry<sup>8</sup>. This estimation leads to a very small vacuum energy of  $\lesssim 10^{-120}$ .

In the rest of this letter we discuss some of other arguments in the literature about the physical reality of very large vacuum energy according to quantum field theory.

### 3 Gravitational coupling of virtual particles

The running mass of elementary particles such as electrons and observed phenomena such as Lamb shift which are due to quantum fluctuations, in another word interaction with virtual particles from vacuum, have been considered as an evidence in favour of a large nonzero vacuum energy obtained in equation (4), see [29] and references therein, and [2, 3]. The claim is that the modification of mass and coupling constants due to loop corrections i.e. exchange of virtual particles, is the evidence of coupling between the vacuum and gravitons, and thereby gravitational interaction of vacuum. Therefore, the observed small nonzero density of dark energy is concluded to be an evidence for the failure of quantum field theory in what concerns its prediction for gravitational interactions. Both scale dependence of mass and Lamb shift are well understood and measured phenomena. Giving the fact that gravitational interaction is proportional to mass/energy, it is evident that the interaction of gravitons with virtual particles from vacuum is included in the mass of elementary particles. To obtain the relation between energy scale and effective mass e.g. in the context of the Quantum Electro-Dynamics (QED) one has to renormalize the theory by removing singularities similar to what obtained in (4). The fact that after this apparently ad hoc operation we obtain relations that are confirmed by experiments, proves that these ad hoc calculations are after all meaningful. If the argument above about a large vacuum energy was true, for the same high energy cutoff, diagrams with larger number of loops should have a larger effect on the effective mass and coupling of electrons at scales less than the cutoff. Thus, they should make a model like QED completely unpredictable. Indeed, the scale dependence of mass and couplings demonstrates that in the context of QED, the effect of virtual particles is controlled by the electromagnetic coupling rather than by the energy-momentum, because with increasing number of loops the small coupling of QED decreases their effect despite the fact that every loop includes a singular momentum integral.

In fact, this effect can be explicitly demonstrated if in analogy with gravity we consider semi-classical electrodynamics in which charged particles are considered to be quantum fields but the electromagnetic field is treated classically. This example is interesting because we know very well both the classical and fully quantic models for electromagnetic interaction in Minkowski spacetime. The semi-classical Maxwell equation can be written as:

$$F^{\mu\nu}_{;\nu}(x) = \langle 0 | j^\mu(x) | 0 \rangle, \quad j^\mu \equiv \bar{\psi} \gamma^\mu \psi \quad (11)$$

To determine the right hand side of (11) at zero-order, we can decompose the fermion field  $\psi$  to modes

---

<sup>8</sup>A heuristic way of reasoning is to say that in a bounded 4-dim empty spacetime of size  $R$ , according to Heisenberg uncertainty principle, the uncertainty on the energy density of quantum fluctuations is  $\sim R^{-4}$ .



similar to (2) with only difference that they anticommute. It can be easily shown that this leads to an integral similar to what is in (4) with a linear singularity due to nonzero anticommutation of modes with the same momentum. Similar to determination of  $T^{\mu\nu}$ , if ordering operator is applied the singularity will be removed. There are also higher order corrections to the vacuum current. If they produce a nonzero current in the right hand side of (11), the classical electromagnetic field would be also nonzero and interacts as an external field with virtual particles. The gravitational analogue of such a back-reaction has been claimed to be the reason for a large vacuum energy density - see diagrams in figure 2 of [2] for examples of interaction between graviton and virtual particles. As explained in the previous paragraph, it is claimed that they generate a very large vacuum energy. Fortunately, in the case of electrodynamics we have a full renormalizable quantum field theory and we know that after regularization and renormalization, the right hand side of equation (11) becomes null, thus QED vacuum has no charge or current. It can be claimed that the neutrality of QED vacuum is due to charge conjugate symmetry. In the neutrino sector of electroweak CP is violated. However, no difference between propagation of neutrinos and anti-neutrinos in vacuum has been observed [30]. Recent claims about observation of a slight difference between neutrinos and anti-neutrinos appearance in short baseline oscillation experiments [31, 32, 33] is most likely related to mixing with non-Standard Model sterile neutrinos, see e.g. [34] or uncertainties of anti-neutrinos initial flux [35, 36], see also below for issues related to electroweak vacuum.

We should remind that the argument in [2] about the interaction of graviton with virtual particles is misleading. In Standard Model mass is generated as a result of interaction (see also the next section for issues concerning the condensation of Higgs). Therefore, it can be considered as the effect of interactions - a delay - in the propagation of a particle. As a classical theory, virtual particles **do not exist** for Einstein gravity. Only in the context of a quantum gravity - which is not available - an interaction between graviton and virtual particles can be envisaged. In Sec. 2.1 we defined the vacuum as a coherent state with vanishing amplitude for all momentum modes. Considering the argument above and the fact that quantum effects of vacuum can be only detected in presence of quantum interactions, we can say that even in the context of quantum field theory vacuum is the empty space without any external effect and what we call vacuum excitations are in fact spontaneous excitation of fields/particles. This interpretation is more consistent with procedure of renormalization and does not leave any ambiguity such as whether graviton interact with vacuum or not.

Another argument in support of the validity of renormalization in curved spacetimes is the fact that predictions of quantum field theory and SM physics are observed and tested in strong gravity field regime around compact objects such as neutron stars, pulsars, and accretion disk of black holes, see e.g. [24]. If the apparent infinities of energy-momentum tensor and other physical quantities before their renormalization were physical, we could not make any prediction for the observed quantities such as modification of emission lines and radiation transfer processes in the accretion disk around black holes in AGNs [37] and compact binary stars [38], the rate of energy loss and slow-down of pulsars by gravitational wave production observed through their radio emission which is produced by electromagnetic interactions in a strong gravity field environment [39], etc.

## 4 Contribution of vacuum expectation value of the Standard Model condensates in dark energy

One of the arguments usually raised in the literature in favor of a large vacuum energy density is the nonzero vacuum expectation value (vev) of Higgs boson and composite fields responsible for chiral symmetry breaking in QCD. The latter case has been recently discussed in [7], and it is argued that the confinement of quarks in hadrons also confines their condensate  $\langle \bar{q}q \rangle$  to the volume occupied by them rather than being homogeneously distributed in space. Therefore, we do not discuss this here. Nonetheless, our arguments in this section

about the contribution of known condensates of elementary particles in dark energy are general and apply to quark condensates as well.

The most important condensate expected in the framework of the Standard Model is Higgs with a nonzero vacuum expectation value expected to be  $\sim 246$  GeV. It is necessary for generating masses of Standard Model particles, in particular the mass of  $W^\pm$  and  $Z_0$  gauge bosons. In addition, it induces the breaking of  $SU(2) \times U(1)$  symmetry at a scale  $\gtrsim 1TeV$ . One should remind that here the word *vacuum* does not mean *empty* space but the minimum of the effective potential of Higgs field(s). The vev can be considered as a classical field, and in analogy with condense matter it is called a condensate. Like other quantities in quantum field theory, it is also subjected to regularization and renormalization. The effective self-interaction potential of the Standard Model Higgs is supposed to have two nonzero minima. The condensation of one of the components of Higgs doublet at one of these minima breaks  $SU(2) \times U(1)$  gauge symmetry and generates mass for  $W^\pm$  and  $Z_0$  bosons. It is usually assumed that the Higgs condensate has a uniform distribution in space. Then, the question arises why the energy density of dark energy - which as a homogeneous density must include the vacuum energy density of Higgs and other scalars such as moduli in string theory - is  $\lesssim 10^{56}$  folds smaller than Higgs vev [2, 3].

Here we argue that this apparent inconsistency is due to the confusion that the energy density of a condensate must be uniform. Conditions for the formation of a Bose-Einstein Condensate (BEC) in a fluid is studied in [40]. The process of condensation of a scalar field in FLRW spacetimes has been discussed in details in [18]. A condensate is defined by decomposing the scalar field:

$$\Phi = \phi + \varphi, \quad \langle \phi \rangle = 0, \quad v \equiv \langle \varphi \rangle \neq 0 \quad (12)$$

It is the component with nonzero expectation value that behaves similar to a classical scalar field - a condensate. In both classical fluid and quantum field theory description of a condensate the amplitude of anisotropies decreases very rapidly for large modes if there is no addition driver to generate large mode fluctuation such as an external force in a fluid, see e.g [41] and references therein, or in the case of quantum scalar fields an interaction with other fields [18]. This observation is analogue to infinite volume condition for symmetry breaking in statistical physics. Note that in contrast to the case of symmetry breaking in which multiple fields and an internal symmetry are assumed, the results in [18] are obtained for single field produced by the decay of another field without any assumption or constraint on their symmetry. The setup in which the scalar has other interactions applies to Higgs condensate. In general, renormalization of the underlying theory induces scale dependence to masses - including Higgs mass - and to couplings. Electroweak interaction is not asymptotically free, therefore couplings increases with energy scale. This means that particles, their interactions, and thereby their condensate are concentrated to short distances.

Another fact to be considered is the feedback of the formation of a condensate on its own evolution and on the evolution of other fields that interact with it. In particular, Higgs condensate interacts with gauge bosons and quarks, and generates mass for them, at least for gauge bosons, thus reduces their dynamism. Moreover, formation of a condensate does not necessarily break symmetries because it can simply uplift the potential. Only when the effective mass becomes negative the  $Z_2$  symmetry of the Higgs potential will break. Therefore, if Higgs is responsible for the breaking of  $SU(2) \times U(1)$  symmetry, it must have significant interactions with other fields, specially fields related to high energy physics - short distance scales - such that the negative mass condition be satisfied. Because the process of condensation is very closely related to interactions, one expects that the condensate distribution has strong correlation with these fields. For Higgs, fields to which it is coupled are: quarks that are confined to atomic distances, high energy physics constituents which are by definition confined to short distances, and gauge bosons  $Z_0$  and  $W^\pm$  which interact with it through  $\propto H^2 A_\mu A^\mu$  term in SM Lagrangian. They have a short free path due to their short lifetime. Detailed study of a simpler case, the condensation of a scalar field produced by the decay of a heavy particle after inflation and during reheating in [18], shows that the condensate can be easily confined by interactions if their couplings to heavy particles are dominant at short distance scales, see equations (84-85) in [18].

Some of Higgs models consider strong interactions for Higgs, either QCD interaction or non-SM interactions with strong couplings [42]. Even if Higgs has only electroweak interaction, according to these arguments the increase of electroweak interaction coupling with scale [29] alone can be sufficient to confine the condensate to short distances. The absence of a large uniform vacuum energy density due to condensation of Higgs or other fundamental scalar fields such as scalar fields in SUSY models or moduli in string theory models - expected to live in higher energy scales - means that their condensates are confined to short distances. Therefore, their effects are included in the renormalized mass and couplings of particles observed at low energies.

We should remind that confinement is also present in superconductivity and superfluidity - the only places where a Higgs phenomenon is actually observed. The analogy between condensation of fundamental fields and what is observed in condense matter is also discussed in [7]. In condense matter the complex scalar is a composite Cooper pair. The interaction that forces electrons to make a spin-zero pair is the effective electromagnetic force of the lattice of ions presented by its pseudo-particle vibrational modes called phonons. The self-interaction of Cooper pairs is usually considered to be  $\phi^4$  type with a global  $Z_2$  symmetry which is broken by the condensate. The latter is confined to where Cooper pairs are present. In this analogy the lattice plays the same role as high energy physics plays for Higgs. Although one of the necessary conditions for formation of a condensate, the symmetry breaking, and the phase transition is a continuous scalar field in an infinite volume, we know that Cooper pairs and their condensate are limited to the volume of a superconductor in the lab. Thus, if the analogy with Higgs condensate is correct, and if the mass and self-interaction of the SM Higgs is related to physics at high energy scales, we expect that its nonzero expectation value is confined to very short distances. Therefore, at low energies its effect is observable only through renormalized properties of particles. In the case of quintessence models for dark energy, the survival of the quintessence field condensate at cosmological scales is a consequence of its very small mass and very weak coupling that leads to the formation of a coherent state which is close to uniform at cosmological distances and survives the expansion of the Universe [18].

Our claim about the confinement of Higgs condensate at high energies can be tested by measuring the energy density of the vacuum at scales close to its expectation value. For instance, considering the interaction between a Higgs doublet and  $SU(2) \times U(1)$  and symmetry breaking by the Standard Model Higgs condensates, Feynman diagrams can include zero, one, or two contribution from the condensate. The latter case generates mass for gauge fields. The lowest order effective interaction between massive gauge bosons and the quantum Higgs field is  $\propto (m_A^2/v) H A^\mu A_\mu$ , see e.g. [43]. Variation of the cross-section of these events with energy scale measures energy dependence or equivalently the spatial extend of the Higgs condensate in space, similar to observation of confinement in strong interaction and scale dependence of QCD coupling. If the signature of Higgs claimed by the LHC Collaboration [44, 45, 46] is confirmed, accumulation of data in near future will make it possible to measure the spatial distribution of the Higgs condensate.

## 5 Outline

We conclude that ordering operation or another regularization/renormalization method are necessary operations to make the definition of energy-momentum tensor consistent with quantum mechanics and quantum field theory principles. They are not ad hoc prescriptions to make the vacuum energy finite when the zero point of energy can be chosen arbitrarily. The apparent ambiguity in the definition of energy is closely related to quantum uncertainties, nonlocal nature of quantum mechanics, and its relation with symmetries. The latter topic needs clarifications that we leave to another work.

We presented a new definition for vacuum as a complete coherent state that includes all states in the Fock space of the Universe with a negligible probability. An observer can only detect a state when it is decohered from this maximally coherent state. It unifies the definition of vacuum as the minimum of the potential

and vacuum as a particle-less state. Such a state can play the role of a *quantum background*. Although it satisfies the usual definition of vacuum state with zero expectation value for all fields, it shows explicitly that quantum vacuum is not as empty as a classical vacuum is. One of its notable properties is its frame-independence. Therefore, in this respect it is unique. It also provides a quantum mechanical discrimination between *real* and *virtual* particles without referring to frame and observer dependent quantities such as energy and momentum. It would be interesting to see whether this definition of vacuum can play the role of a background in some of quantum gravity models in which there is no apparent background.

Using this vacuum state and analogy between semi-classical gravity and semi-classical electrodynamics, we showed that the apparent large or infinite energy density of vacuum is not a physical observable and can be safely regularized. We discussed the issue of Higgs large vacuum expectation and argued that it is confined to scales of the order of electroweak symmetry breaking scale. These explanations persuade us to reconsider the interpretation of dark energy as the *energy of vacuum*, and encourage explanations based on either condensation of fields - quintessence models - or a modification of gravity. Notably, in some classes of quintessence models the condensate density can be very uniform and has an equation of state very close to a cosmological constant [47]. Only observations of additional evidence such as interacting/decaying dark matter, and interaction in the dark sector can discriminate these models from a genuine cosmological constant [50].

## References

- [1] E.J. Copland, M. Sami, & S. Tsujikama, *Int. J. Mod. Phys. D* **15**, (2006) 1753 [hep-th/0603057].
- [2] J. Polchinski [hep-th/0603249].
- [3] R. Bousso [arXiv:1203.0307].
- [4] A.A. Belavin, *et al.*, *Phys. Lett. B* **59**, (1975) 85.
- [5] G. t'Hooft, *Phys. Rev. D* **14**, (1976) 3432.
- [6] A. Casher & L. Susskind, *Phys. Rev. D* **9**, (1974) 436.
- [7] S.J. Brodsky & R. Shrock, *Pub. Nation. Acad. Sci.*, (2011) 45.
- [8] M. M. Caldarelli, *Nucl. Phys. B* **549**, (1999) 499, [hep-th/9809144].
- [9] S. Weinberg, *Rep. Mod. Phys.* **91**, (1989) 1.
- [10] N.D. Birrell & P.C.W. Davis, *Quanyum fields in curved space*, Cambridge Univ. Press, (1986).
- [11] E. Witten, [hep-ph/0002297].
- [12] J. Kumar, *IMA* **21**, (2006) 3441 [hep-th/0601053] (review).
- [13] S. Weinberg, *PRL* **59**, (1987) 2607.
- [14] A. Vilenkin, *Phys. Rev. Lett.* **74**, (1995) 846.
- [15] K. Eppley, & E. Hannah E., *Found. Phys.* **7**, (1977) 51.
- [16] H. Ziaepour, *J. Phys.: Conf. Series* **174**, (2009) 012027 [arXiv:0901.4634].
- [17] S. Matsumoto and T. Moroi *Phys. Rev. D* **77**, (2008) 045014, arXiv:0709.4338.

- [18] H. Ziaeeepour, *Phys. Rev. D* **81**, (2010) 103526 [arXiv:1003.2996]
- [19] M. Kira, G. Khitrova, & H.M. Gibbs, *Nature Mater.* **5**, (2006) 523.
- [20] H.B.G. Casimir, *Proc. K. Ned. Akad. Wet*, **51** (1948) 793.
- [21] S.K. Lamoreaux, *Phys. Rev. Lett.* **78**, (1997) 5.
- [22] G.L. Klimchitskaya, *et al.*, *Int. J. Mod. Phys. A* , (2005) 1.
- [23] Taylor, J.H., Fowler, L.A., & Weisberg, J.M., *Nature* **277**, (1979) 437.
- [24] S. Vaughan, A. C. Fabian, *MNRAS* **341**, (2003) 496 [astro-ph/0301172].
- [25] B. Allen, *Phys. Rev. D* **32**, (1985) 3136.
- [26] T.S. Bunch & P.C.W. Davis, *Proc.Roy.Soc.Lond.* **360**, (1978) 117.
- [27] M.B. Einhorn & F. Larson [hep-ph/0305056].
- [28] Wald, R.M., The University of Chicago Press, (1984).
- [29] S. Weinberg, *The Quantum Theory of Fields* vol. 2, Cambridge University Press (1997).
- [30] A.A. Aguilar-Arevalo, *et al.*, [arXiv:1109.3480].
- [31] A.A. Aguilar-Arevalo, *et al.*, *Phys. Rev. D* **64**, (2001) 112007 [hep-ex/0104049].
- [32] A.A. Aguilar-Arevalo, *et al.*, *Phys. Rev. Lett.* **98**, (2007) 231801 [arXiv:0704.1500].
- [33] A.A. Aguilar-Arevalo, *et al.*, *Phys. Rev. Lett.* **105**, (2010) 181801 [arXiv:1007.1150].
- [34] E. Akhmedov, T. Schwetz, *J. High Ener. Phys.* **1010**, (2010) 115 [arXiv:1007.4171].
- [35] A.A. Aguilar-Arevalo, *et al.*, *Phys. Rev. D* **84**, (2011) 072005 [arXiv:1102.1964].
- [36] G. Mention, M. Fechner, Th. Lasserre, Th. A. Mueller, D. Lhuillier, M. Cribier, A. Letourneau, *Phys. Rev. D* **83**, (2011) 073006 [arXiv:1101.2755].
- [37] J. Wilms, *et al.*, *MNRAS* **328**, (2001) L27 [astro-ph/0110520].
- [38] L. Angelini, N.E. White, *ApJ*. **586**, (2003) L71 [astro-ph/0302315].
- [39] S. Zane, *et al.*, *MNRAS* **334**, (2002) 345 [astro-ph/0203105].
- [40] O. Penrose & L. Onsager, *Phys. Rev.* **104**, (1956) 576.
- [41] M.L. Chiofalo & M.P. Tosi, *Europhys. Lett.* **56**, (2001) 326.
- [42] G.F. Giudice, C. Grojean, A. Pomarol, R. Rattazzi, *J. High Ener. Phys.* **0706**, (2007) 045 [hep-ph/0703164].
- [43] I. Low, J. Lykken, *J. High Ener. Phys.* **10**, (2010) 053 [arXiv:1005.0872].
- [44] ATLAS Collaboration, *Phys. Lett. B* **710**, (2012) 49.
- [45] ATLAS Collaboration, *Phys. Rev. Lett.* **108**, (2012) 111803 [arXiv:1202.1414].
- [46] ATLAS Collaboration, *Phys. Lett. B* **710**, (2012) 383 [arXiv:1202.1415].



- [47] H. Ziaeeepour, *Phys. Rev. D* **69**, (2004) 063512 [astro-ph/0308515].
- [48] H. Ziaeeepour, *Mod. Phys. Lett. A* **22**, (2007) 1569 [astro-ph/0702519].
- [49] H. Ziaeeepour, *AIP Conf.Proc.* **957**, (2007) 453 [arXiv:0709.0115],
- [50] H. Ziaeeepour, to be published in *Phys. Rev. D*, 30 July (2012) [arXiv:1112.6025].